

# A STATISTICAL METHOD FOR FAIL-SAFE DESIGN WITH RESPECT TO AIRCRAFT FATIGUE

The Reduced Static Strength Criterion Applied to a  
Diffuse Wing Structure

By BO. K. O. LUNDBERG and SIGGE EGGWERTZ

*Summary*—A fail-safe aircraft structure containing a fatigue crack can still carry a certain reduced static loading. As the crack propagates and the strength of the structure decreases, the probability of collapse due to a heavy gust or manoeuvre load increases rapidly with time. It is possible to limit the probability of collapse to a required low level by regular inspections. A method of calculating the probability of collapse is given for a diffuse wing structure, taking into account the growing probability with service time of crack initiation as well as crack propagation and inspection frequency. The method is applied to realistic numerical examples.

## 1. INTRODUCTION

SINCE 1954 the need for a quantitative statistical approach to the fatigue problem has been stressed by the FFA in a number of publications<sup>(1-6)</sup> indicating some general methods of dealing with the problem both for fail-safe and safe-life designs. In one of the latest of these papers<sup>(5)</sup>, the basic features of and the main reasons for such an approach are summarized. Briefly, it might be stated that as design for fatigue for a considerable time has been acknowledged as a fundamental safety question of equal importance as the static strength case, and as fatigue which to a large, often dominating extent determines the structural weight, it seems unsatisfactory that fatigue should still only be treated qualitatively in vague terms.

The reluctance to accept a quantitative statistical approach is probably not due to objections of a fundamental nature against statistical treatments as such of technical problems in aviation, since statistical methods have already been applied in the airworthiness requirements. For instance, the present climb performance code was originally based on statistical evaluation of the incident rate due to engine failures.

A main objection against the statistical approach to the fatigue problem was, for some time, that adoption of the fail-safe design principle was considered by many to make a statistical evaluation of the risk of

structural failure unnecessary. Such an evaluation was maintained to be required only for safe-life designs because of the scatter in fatigue properties, whereas the risks of complete structural failure due to this scatter was considered to be eliminated with fail-safe designs. It was, however, soon generally recognized that inspection at appropriate intervals is a necessary attribute to fail-safe designs; with no or inadequate inspections a fail-safe design might be quite unsafe. Obviously, the frequency of inspections largely determines the probability of collapse of a fail-safe structure, which implies that the safety of such structures is of a statistical nature. This makes the statistical approach at least desirable, in the authors' opinion necessary.

However, there remains another objection against statistical approach to the design of fail-safe structures with respect to fatigue and that is a questioning whether such an approach is really feasible at the present state of the art. Undoubtedly, there are reasons for this objection. Some rather detailed mathematical principles of statistical analysis of fail-safe structures with regard to the probability of collapse due primarily to fatigue have been outlined in a number of contributions<sup>(4, 7, 8)</sup>, but so far no sufficiently adequate procedure has been established; either the methods are presented based on unduly simplified assumptions or they are incomplete in one way or another. For instance, Ferrari and co-authors in a rather advanced study<sup>(8)</sup> have presented a statistical method for the determination of the required fail-safe strength, i.e. the residual strength after a certain "fail-safe damage". This method, however, neglects the influence of regular inspection at predetermined intervals and therefore the resulting fail-safe strength will be unnecessarily high for a required maximum probability of collapse. At the FFA the so-called Critical Number Criterion has been treated, i.e. the probability of two more partial failures—with particular reference to a multi-spar wing structure with heavy spar booms—occurring in the same inspection interval and leading to complete failure under the influence merely of normal (1 g) loads<sup>(4)</sup>. Obviously this method is incomplete as a basis for design as one has to consider also the probability of exceeding the reduced static strength after merely one partial failure. Furthermore, the simplifying assumption made of zero or very short crack propagation time might be unduly conservative unless the design is such that a crack in a spar boom cannot be detected with certainty, even at thorough inspections, before it has completely fractured the boom.

The said might suffice to indicate the nature of the difficulties involved when dealing with fail-safe structures on a statistical basis. A vast research field is envisaged that has to be explored to a large extent before adequate statistical methods can be employed by designers. Due to the

complexity of the problems involved and the great number of fundamentally different structural design principles conceivable, the exploration will have to be conducted step-wise by treating various types of design separately and by making the assumptions in the theories less and less simplified, i.e. more and more approaching actual conditions.

This paper will deal with a fail-safe designed wing structure of the diffuse type considering the case of reduced static strength due to progressive propagation of fatigue cracks, which case is termed the "Reduced Static Strength Criterion". Both the growing probability with service time of crack initiation and inspections at predetermined intervals are taken into account.

## 2. DEFINITION OF THE SAFE-LIFE AND FAIL-SAFE CONCEPTS

Although the basic difference between safe-life and fail-safe structural parts or assemblies has been generally appreciated for some time, the definitions used for the two concepts in various national standards and in the ICAO airworthiness standards differ appreciably. The following definitions are proposed:

A safe-life structure is one for which a certain demanded low probability of collapse during a specified reference service life—the "safe limit life"—is assessed by due consideration for the scatter in fatigue properties of the structure and other uncertainties.

A fail-safe structure is one for which a certain demanded low probability of collapse during a specified reference service life—the "fail-safe limit life"—is assessed by a combination of (a) design features, such that one or more conceivable partial failures or cracks due to fatigue merely lead to a limited reduction of static strength or impairment of other properties essential to airworthiness, with (b) inspections, or automatic warning, ascertaining the partial failures or cracks to be detected. The specified fail-safe limit life varies in principle with the inspection intervals and procedures implying that the limit life can be extended by applying more frequent or stringent inspections.

Apart from formal differences, these definitions are the same as those proposed to ICAO in 1959<sup>(6)</sup>. Compared with the corresponding definitions suggested the year before<sup>(3)</sup>, they differ in two respects, the most important one being that the fail-safe type of structures is also intimately connected with a life limitation, although this limitation varies with the frequency of inspections. The dependence of the safety of fail-safe structures on their service life was shown in<sup>(1)</sup> and <sup>(7)</sup>, and the importance of this fact was particularly stressed in<sup>(3)</sup>. In<sup>(4)</sup> it was shown quantitatively

that with respect to the Critical Number Criterion, the probability distribution for fail-safe structures of a multi-spar type is quite similar to that for a safe-life structure, it being of particular importance that the probability of collapse also for fail-safe structures increases rapidly as the service life approaches the level where the individual structural elements reach their mean fatigue lives. In the following it is indicated that the same applies with a diffuse type of fail-safe structure considering the Reduced Static Strength Criterion.

In view of the said rapid increase with service life of the probability of collapse of fail-safe structures it seems important to include the "fail-safe limit life" concept in the definition of fail-safe structures. This would counteract the misconception that fail-safe structures, merely by virtue of the design principle, can be safely used more or less indefinitely.

The other modification introduced in the two definitions suggested above is the word "reference" as an attribute to "service life". In<sup>(6)</sup> the following definition was proposed:

"The reference service life is the fictitious current life in hours of flight of an aircraft assumed to operate in defined reference operational conditions. These should be chosen (by the designer) so as to represent average operational conditions—for the aircraft as a whole (flight plan) and for its structural parts—that are typical for the intended use of the aircraft type".

The actual service life of the aircraft and/or its assemblies and parts should be transferred into reference service life on the basis of approximately the same fatigue damage being experienced by the structure<sup>(2)</sup>. This transfer (and keeping a log over the reference service life) should be made by the operator according to relationships developed by the designer for the various components of the aircraft.

### 3. ACCEPTABLE PROBABILITY OF FAILURE.

Based on estimates of the development of commercial aviation during the next few decades and the desirability of reducing the numbers of fatal accidents due to structural fatigue to an extent at which they are practically unheard of, it was proposed in<sup>(1)</sup> and<sup>(3)</sup> that the general, long-term safety goal, in particular for commercial aviation, with respect to this source of accidents should be a failure rate not exceeding a value of  $10^{-9}$  per hour of flight. For practical reasons it seems preferable to base the design of an aircraft on a maximum "limit probability" of fatal failure during the limit life of the aircraft, and a figure of  $P_L = 10^{-5}$  has been proposed<sup>(5,6)</sup>.

The two figures will probably be subjected to further discussion before internationally agreed on. In view of the tremendous growth of non-commercial civil aviation to be expected in the future—not least due to the forthcoming “V/STOL-Age”—and the fact that accidents in one field of aviation, such as executive flying, always more or less affects public confidence in other fields, thus also commercial aviation, the said figures now seem too high rather than too low; a decrease by a factor of 10 is well worth considering, implying  $P_L = 10^{-6}$ .

Whatever figure for the limit probability of the indicated order of magnitude is eventually agreed on as the basic fatigue design requirement, the magnitude will be so small that the following approximation for the resulting probability,  $P_R$ , is valid with sufficient accuracy:

$$P_R = \sum P_v \leq P_L \quad (1)$$

where  $P_v$  is the “part probability” of any one occurrence, i.e. a local structural failure leading to collapse of the structure. It should be observed that all the part probabilities,  $P_v$ , have to be “statistically independent” in relation to each other. If there are a “group” of probabilities of failure which are not independent, their resulting probability has to be assessed and then treated as one of the part probabilities in (1).

The relationship (1) is the basis for the “sharing principle”, suggested in <sup>(3)</sup>. The practical implication of this principle as a design guidance is:

- (a) that the admitted limit probability is divided into two principal shares, one for all the safe-life elements and the other for all the fail-safe assemblies, provided, of course, that both design principles are applied,

and

- (b) that each of these principal shares are subdivided into portions for all independent structural assemblies, elements or sections of elements or even local spots of elements, such as rivets.

How far one should go into the subdividing (b) is, in principle, a matter of choice; the important thing is that it corresponds to the test results on which the statistical evaluations have to be based. In practice the degree of subdivision and, thus, the choice of the tests to be made is an optimum question with regard to cost. If, for instance, the evaluation is based on a sufficient number of tests with complete assemblies, comprising half the wing, then there is no point in further subdivision, but such tests are normally much too expensive. A limited number of tests with wing assemblies might, however, be statistically sufficient if the test results can be supported by large sample size tests with wings which are quite similar both with regard to detail design, material and stress levels

to the one under design, but even so such a procedure might often prove to be prohibitively expensive.

With regard to aircraft wings, it is believed that the optimum procedure often would be tests with a number of wing sections, which, in view of stress levels and detail design, are judged to be fatigue sensitive. Even such tests are, however, expensive when it is desirable to support the results of a limited number of tests with large sample size tests conducted elsewhere with similar wing sections.

The method developed in this paper is primarily intended for evaluating the probability of complete failure in a chordwise section of an aircraft wing. To get an idea of the order of magnitude of the limit probability,  $P_{LS}$ , that can be allowed for collapse of such a wing section, a few observations might be made. Assuming that there are one or more safe-life parts in the aircraft structure, the principal share for all the fail-safe assemblies might be chosen anywhere from say 0.1 to 0.9 of  $P_L$ . If the weight of the safe-life parts is appreciable, it is usually efficient to allot to them a big share—say 0.9—considering the difficulties to ascertain a low probability of failure for safe-life parts with an acceptable confidence. A weight-efficient subdividing of the fail-safe probability share in portions for all the fail-safe assemblies usually implies a rather big portion for the wing, e.g. 0.2 to 0.5, because the wing normally constitutes a considerable portion of the total structural weight and is a rather complex structure with regard to statistical treatment. Finally, the subdividing of the probability portion allotted to the wing is dependent on the number of statistically independent regions or sections that the wing comprises. If we, for instance, assume that there are as many as 20 such sections and that all are “statistically equivalent”, e.g. have the same mean life and standard deviation, then each section would be allotted 0.05 of the probability for the whole wing.

It follows that the limit probability for a wing section,  $P_{LS}$ , might range from about  $10^{-9}$  to about  $10^{-6}$ . In most cases it seems reasonable to assume that  $P_{LS}$  has to be of the order of  $10^{-8}$  to  $10^{-7}$ .

#### 4. DEFINITION OF DIFFUSE STRUCTURES

Fail-safe properties for an aircraft wing are usually obtained either by a distinct multi-load-path arrangement—comprising, in most cases, three or more spars—or by a so-called diffuse structure or by a mixture or compromise between the two. A typical Diffuse Structure is illustrated in Fig. 1. It is characterized by a fairly thick skin, as a rule stiffened by a large number of not too heavy stringers, and by a number of spars, usually two or three, the booms of which have comparatively small cross-

sectional areas. The main part of the bending moment is thus taken by the skin or skin-stringer combination.

Considering first the "bending material", a crack may be initiated at some stress concentration either in the skin or in a boom or a stiffener but, in general, the crack will not cause failure until it has propagated to a considerable length through the skin. There is evidence<sup>(8)</sup> that for diffuse wing structures the reduction in static strength caused by prop-

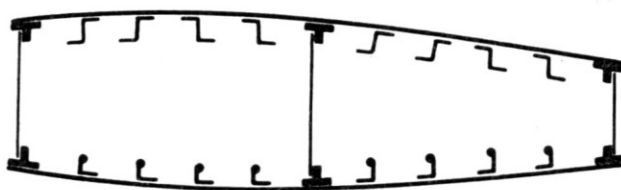


FIG. 1. Schematic illustration of section of a three-spar wing with a stiffened load-carrying skin, an example of a diffuse structure.

agation of a crack due to variable loads usually follows a fairly smooth continuous curve if plotted versus number of load cycles. This curve might be either downward concave or downward convex or even an approximate a straight line. There might, however, often be more or less pronounced discontinuities in the curve depending on the detail design; usually each stringer or boom acts as a "crack delayer". It is proposed that a diffuse structure simply be defined as one having crack-propagation/strength reduction properties which can be approximated to a continuous relationship. Although the method developed in this paper is based on the simple assumption of a straight line relationship, it can easily be extended to other continuous forms of the curve. The method is thus applicable not only for diffuse wings but also for many other structural assemblies elements, such as stabilizers, fins and control surfaces, provided that they comply with the said definition.

The "shear material" is also of importance when dealing statistically with a wing structure, the bending material of which is of the diffuse type. The possibility of a crack in the web might have to be treated separately, because the strength reduction due to the propagation of such a crack might follow a different pattern from that of a crack propagating in the skin. If this is the case, it must be observed that the "total probability" of collapse of the section, considering both the webs and the skin, should not exceed  $P_{LS}$ . If there are only one or two spar-webs, it might be necessary, or at least advisable, to treat the webs as safe-life components. If there are three or more webs, it might be feasible to overcome the difficulty due to the possibility of cracks in the webs by designing them—

primarily by choosing a low stress level—for a probability of crack initiation that is appreciably less than that for the skin. Otherwise it must be observed that the probabilities of collapse of the section due to cracks in the skin and in the webs can hardly be considered as statistically independent because of the redistribution of the loads when the skin or the webs have partially been fractured.

##### 5. THE REDUCED STATIC STRENGTH CRITERION

The design of a fail-safe structure seems to be performed at present by experimental or theoretical verification that the structure can still carry a load of a certain magnitude, the Fail-Safe Load, after the structure has suffered a certain "Fail-Safe-Damage", such as "an obvious partial failure" of a single structural element or the development of a crack in the skin of a considerable length. It is thus assumed that the fail-safe damage will be discovered with certainty at the latest before the next flight. Obviously such detection is equivalent to—or a form of—"automatic warning" referred to in the definition above of fail-safe structures.

From the moment a crack starts to reduce the strength of the structure until the partial failure or crack is discovered, a probability of failure, in the first place due to heavy gust loads, is obviously building up which is considerably larger than the probability of static failure due to gusts on an undamaged structure during the same length of service time. By imposing an upper limit on the said increased probability of failure a Fail-Safe Load Criterion can be defined with a quantitative statistical implication.

A main difficulty with the fail-safe-load approach is, obviously, the determination of the fail-safe-damage that will be detected with certainty. This depends mainly on the design, but to some extent also on the standard with regard to daily maintenance, etc. If the design is such that the fail-safe-damage must be quite considerable to enable detection under normal maintenance conditions, then the residual strength will be much lower than the original ultimate strength. For compliance with a quantitative statistical requirement a great reduction of the original strength after occurrence of the fail-safe-damage necessitates a considerable extra margin in the design load factor, implying a great weight penalty. The possibilities of complying with a required low probability of collapse might, however, be improved by regular inspections, at which fatigue damage much less than the fail-safe-damage is detected, but then the verification of the fail-safe strength is no longer the only condition for attaining an acceptable safety level; regular inspections might well be the most important provision for ensuring safety.



If, thus, emphasis is laid on inspections, the concept of fail-safe load does not adequately cover the real mechanism that usually governs the safety of fail-safe structures. It is proposed that the term "Reduced Static Strength" is introduced instead, as covering the whole picture of a successively weakening structure after a crack has developed to such an extent that the static strength starts to decrease. The corresponding Reduced Static Strength Criterion would then imply a quantitative statistical treatment of the probability of failure of the structure, considering the strength reduction due to the appearance and propagation of a major crack.

For the following application of this criterion to a diffuse wing section it is assumed that inspections at regular predetermined intervals are made and that all partial failures and cracks equal to or exceeding a certain small "detectable length"—for instance around a quarter of an inch<sup>(9)</sup>—are detected at such inspections. It is also assumed that cracks which have just reached the detectable length do not reduce the static strength. It might consequently be assumed that the static strength of the structure is equal to the original strength immediately after the inspection, since all cracks of detectable length or longer will then be repaired. During an inspection interval, however, a crack exceeding the detectable length may develop and cause an increasing reduction of the static strengths. In each inspection interval the risk of failure due to a heavy gust is thus increased in comparison with the risk of static failure of the undamaged structure. The probability of failure during one interval is, furthermore, as a rule greater than during any one of the preceding intervals as the risk of crack initiation increases with service time. The sum for the various inspection intervals of the probabilities of complete failure of the wing section should now be limited to a value which at the most should equal  $P_{LS}$ .

It should be observed that the concept of a certain distinct fail-safe load is quite compatible with the reduced static strength concept as a supplementary safeguard, provided that the design is such that the corresponding fail-safe-damage will be detected with certainty also between inspections. This would obviously reduce the probability of collapse compared with the case where such a crack is not detected until the next inspection. This "Supplementary Fail-Safe-Load" condition has not been taken into account in the method developed, although it seems quite possible to do so.

There is a certain risk that two or more cracks might be initiated during the same inspection interval. The probability of two cracks occurring in the same section is, of course, much lower than that of one crack appearing. On the other hand the situation is likely to be more severe with two or more cracks as these will probably cause a higher crack propagation rate. This increased probability of collapse is, however, not treated in the

method of Chapter 6. This method does not either take into account the before-mentioned possibility of one or more cracks in the spar-webs. It might be pointed out, however, that these two simplifications on the "unsafe side" are counter-balanced by the conservative assumption that no cracks will be discovered between the regular inspections, not even if they reach a length that would imply "fail-safe-damage".

## 6. A METHOD OF CALCULATING THE PROBABILITY OF COLLAPSE

### 6.1. *General*

The method developed presupposes knowledge of (a) the fatigue and crack propagation properties of the structure—in particular a diffuse wing section—under the spectrum of loads assumed to occur with reference to operational conditions, and of (b) the spectrum of heavy gust loads, in average thunderstorm turbulence. The mathematical background for the method may be found in Appendix A.

In the application of the method the fatigue and crack propagation properties of a structure to be designed or checked, are, in principle, most reliably assessed by tests with a great number of nominally identical specimens of the structure so that the scatter can be determined with sufficient accuracy. In practice usually only a fairly small number of full-scale tests can be made. These will then mainly have the purpose of indicating the mean values, whereas they only can give a rough estimation of the scatter parameters. Therefore these parameters have to be assumed on the basis of results from laboratory tests of similar full-scale structures or representative smaller structural specimens or even elements.

In the following presentation of the method the order of magnitude of the variables involved is discussed and exemplified with reference to transport aircraft. In addition, a numerical example is presented indicating how the method can be used for an actual design so as to comply with a certain permitted limit probability for a wing section,  $P_{LS}$ . An appropriate choice of the stress level is emphasized as the main tool of the designer to meet the required  $P_{LS}$  if the fatigue quality as such of the structure cannot be improved and the inspection interval cannot be shortened for practical reasons.

### 6.2. *Crack initiation*

As indicated above and in the Appendix the "initiation" of a crack is defined as the stage when a crack has reached a certain detectable length. This has to be chosen by the designer on the basis of, *inter alia*, the inspection methods that will be demanded. The fatigue properties of the structure in terms of crack initiation can be assessed either from cumulative damage calculations with results on fatigue tests at constant amplitudes,

$S-N$  tests, or by performing spectrum fatigue tests with a load spectrum corresponding to reference operational conditions. The advantages and disadvantages of these procedures have been discussed by many authors and it has also been pointed out how a few spectrum tests may be supported by computations based on  $S-N$  testing<sup>(2, 10)</sup>.

It is assumed here that the fatigue life in hours until crack initiation has a log-normal distribution. The probability of a crack occurring before  $T$  hours of flight is thus

$$P_c = \phi\left(\frac{\log T - m}{\sigma}\right) \quad (2)$$

where  $m$  and  $\sigma$  are the mean value and the standard deviation of  $\log T$ .

From the investigations of crack propagation which have been performed on wing structures both at constant and variable load amplitudes<sup>(9, 11, 12)</sup>, it may be concluded that most of the scatter in fatigue testing

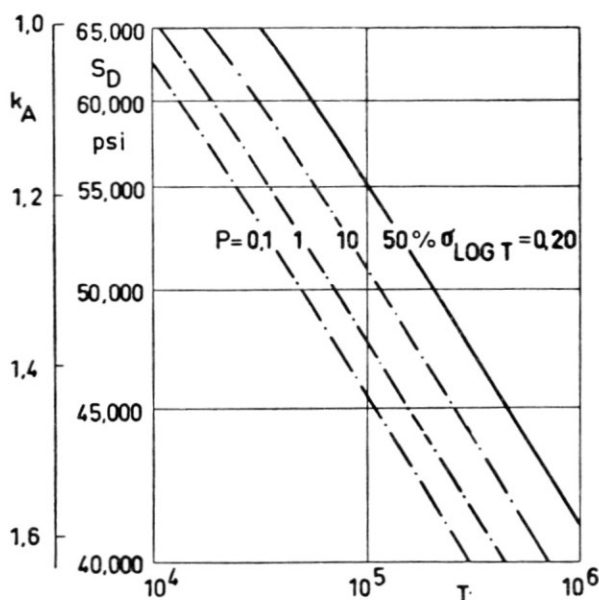


FIG. 2. Example of ultimate design stress versus hours of flight until a fatigue crack appears with a probability  $P$  from 0.1 to 50%. Material 24S-T, stress concentration  $K_t=4$ .

is involved in the period until crack initiation, whereas the crack development time is a comparatively constant period. The mean value, and probably to some extent also the standard deviation, depend on the stress level of the structure, which can be expressed by the ultimate design stress  $S_D$  or by the fatigue factor  $k_A$ , which is a measure of the increase in structural area called for by fatigue considerations<sup>(1, 2)</sup>.

If the design is such that it just meets the static strength requirements ( $k_A = 1.0$ ), which are assumed to call for an ultimate design load factor of 3.75, the logarithmic mean value

$$m = ({}^{10}\log T)_{50}$$

for the initiation of the first crack should be somewhere between 4 and 5 for an ordinary well-designed wing structure of 24S-T at reference operational conditions. The corresponding number of hours of flight, denoted here by  $T_{50}$ , is thus  $10^4$ – $10^5$ . With regard to the standard deviation of  $\log_{10} T$  a study of constant amplitude tests on entire wing structures indicates a value of 0.2–0.3<sup>(12)</sup>. Spectrum tests might give still higher standard deviations<sup>(9)</sup>.

Figure 2 gives an indication of when the first crack might appear in a conventional 24S-T structure at various ultimate design stresses. The median curve ( $P = 50\%$ ) has been computed from test data<sup>(13)</sup> on a coupon with two edge-cut notches (stress concentration factor  $K_t = 4$ ) with the linear cumulative damage theory, assuming a path ratio of 0.1 for the normal turbulence spectrum<sup>(1)</sup>.

### 6.3. Crack development and reduction of ultimate strength

A number of investigations of crack propagation rate and residual static strength of structural elements, as well as of entire wing structures, have been published in the last few years<sup>(9, 11, 12, 14–16)</sup>. From the theories advanced in combination with some testing, it should be possible to determine for a structure under consideration the ultimate static strength as a function of the time elapsed since a crack has reached the detectable length, the crack propagation time,  $t$ . In a general treatment of the problem without reference to a specific structure it seems appropriate to assume a linear relationship between residual strength and crack propagation time.

If  $S_u$  is the original ultimate strength and  $S_m$  the mean load, then  $S = S_u - S_m$  is the original static margin in 1 *g* level flight. The residual static margin,  $t$  hours of flight after a crack has occurred, is denoted by  $S_t$ . The linear relationship implies

$$S_t = S \left( 1 - \frac{t}{R} \right)$$

or

$$s_t = \frac{S_t}{S} = 1 - \frac{t}{R} \quad (3)$$

where  $R$  is the "crack propagation parameter", if the relationship is assumed to hold until the static margin has vanished. Since the crack will

usually be discovered considerably before the time  $t = R$  it is not necessary that equation (3) is valid for more than a fraction of the whole crack propagation time. The parameter  $R$ , which varies with the stress level, seem to be of the order of magnitude of 10,000 with a probable range of variation from 2000 to 20,000 hours, if the ultimate design load factor is around 3.75.

In the statistical analysis of Appendix A, equation (3) has been used in order to simplify the mathematical expressions. The numerical evaluations which have been made would only be slightly complicated, however, by a nonlinear relationship.

#### 6.4. Spectrum of heavy gust loads

It seems appropriate to separate the gust loads on an aircraft wing into two categories, normal (clear) air gusts and thunderstorm gusts<sup>(8,17)</sup>. When considering the fatigue damage of the structure it is usually enough to take only the normal air gust spectrum into account, while the static failure of a cracked or uncracked structure is as a rule caused by the heavy gusts of the thunderstorm spectrum. This latter spectrum may be approximated by a straight line in a semi-log plot<sup>(1)</sup>

$$H = H_0 e^{-hs_t} \quad (4)$$

where  $H$  is the number of gust cycles per hour which exceed the relative residual static margin defined in equation (3).

For modern civil transport aircraft the parameter  $H_0$  may be expected to range from 0.1–0.3<sup>(8)</sup>, being dependent mainly on the flight plan. In this paper all calculations are based on an average value  $H_0 = 0.2$ , corresponding to reference operational conditions.

The parameter  $h$  depends on the design stress level, the relative equivalent air speed and the relative weight of the aircraft<sup>(1,18)</sup>. For an aircraft with an ultimate design load factor of 3.75, flying at a reduced speed of 0.75  $V_C$  in rough air, the value of  $h$  would be around 20. A decrease in design stress level results in a linear increase in  $h$ .

#### 6.5. Probability of collapse including inspection

It is assumed that the limit life  $T_L$  of the structure is divided into  $n$  inspection intervals. During an arbitrary interval the probability of collapse is virtually a combination of the probabilities of crack initiation and the crack leading to static failure after crack propagation. The resulting probability may be written as an integral (see Appendix A)

$$\int_{T_v-1}^{T_v} \frac{dP_C}{dT} P(T_v - T) dT$$

where  $dP_C/dT$  is the probability density of a crack appearing, and  $P(T_v-T)$  the probability of failure of the cracked structure during the time  $T_v-T$  due to a heavy gust. The total probability of collapse of the section during the whole limit life is the sum over all the  $n$  inspection intervals

$$P_S = \sum_{v=1}^u \int_{T_{v-1}}^{T_v} \frac{dP_C}{dT} P(T_v-T) dT \quad (5)$$

The probability  $P(T_v-T)$  may be obtained from the formula (see Appendix A and<sup>(8)</sup>)

$$P(T_v-T) = 1 - e^{-\frac{RH_0}{h} e^{-h} [1 - e^{h(T_v-T)/R}]} \quad (6)$$

Equation (6) presumes that any existing crack is discovered and repaired at each inspection, so that the residual static margin is equal to the original static margin, i.e.  $s_i = 1$ , immediately after an inspection.

The probability density  $dP_C/dT$  can be obtained by derivation of equation (2). This procedure neglects the effect of repairs with respect to the fatigue properties. Strictly, each repair carried out in the structure should result in a jump downwards in the density function, since part of the section is replaced by material which has not been subjected to fatigue. It is possible to take the effect of repair into account in the method presented, but lacking experimental evidence as to the magnitude of the discontinuities, these have been omitted in the numerical calculations.

In equation (5) the length of the inspection intervals may vary in an arbitrary manner. If it is assumed to be constant with a value  $t_i$ , i.e.

$$t_i = T_v - T_{v-1}$$

the numerical evaluation of the formula is greatly simplified.

### 6.6. Numerical examples

The probability  $P_S$  of total failure of the wing section according to equation (5) can be integrated by means of approximate numerical methods. Using numerical values of the probability functions of crack initiation and the crack leading to static failure, obtained from equations (2) and (6), and dividing the inspection intervals into a number of steps which are small enough to provide a sufficient accuracy (see Appendix A), an introductory example was first calculated on an electronic digital computer assuming the following values of the parameters.

Crack initiation	$T_{50} = 50,000$	$\sigma = 0.20$
Crack development	$R = 10,000$	
Gust spectrum	$H_0 = 0.20$	$h = 20$

The calculations were carried out for a range of limit life  $T_L$  from 5000 to 60,000 hours with different numbers of equally long inspection intervals  $n$  up to 12. The resulting probability of collapse  $P_S$  has been plotted in Fig. 3, where the assumed probability function  $P_C$  has also

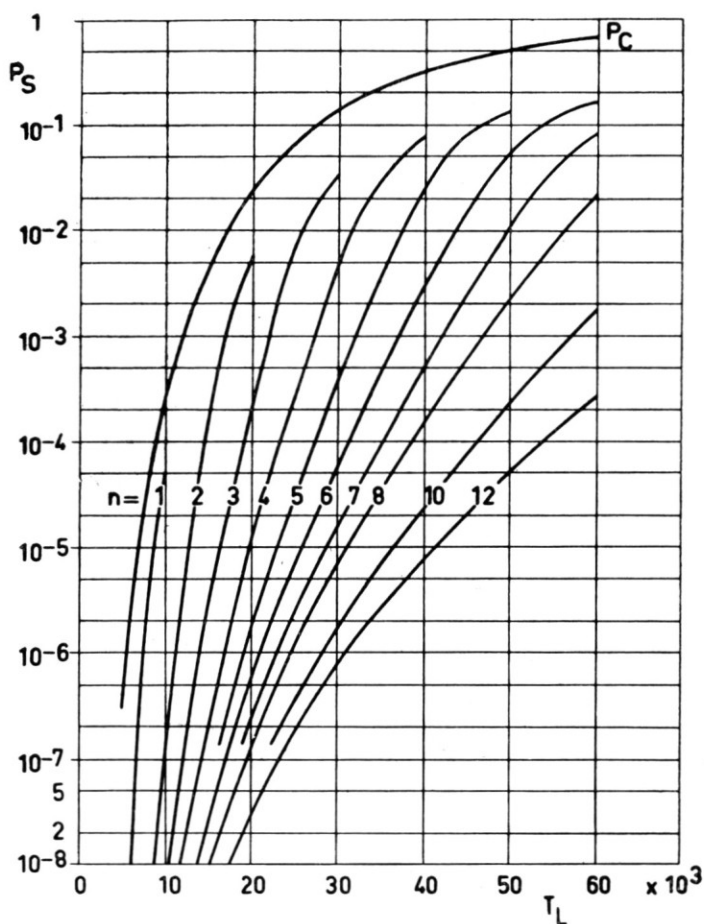


FIG. 3. Probability of collapse  $P_S$  when inspection is performed with a number of  $n$  intervals during service life  $T_L$ .  $P_C$  is the probability of crack initiation. Parameters assumed:

Crack initiation  $T_{50} = 50,000$   $\sigma = 0.20$   
 Crack development  $R = 10,000$   
 Gust spectrum  $H_0 = 0.20$   $h = 20$

been introduced for comparison. One inspection interval ( $n = 1$ ) implies that the aircraft would fly from the beginning till the end of the limit life without any intermediate inspection. The section would then by definition be a safe-life structure. One intermediate inspection ( $n = 2$ )

will cause a considerable reduction in the probability of collapse and a further reduction is obtained with  $n = 3, 4...$

Figure 3 illustrates the important fact that the probability functions for fail-safe structures ( $n \geq 2$ ) have a form rather similar to that of a

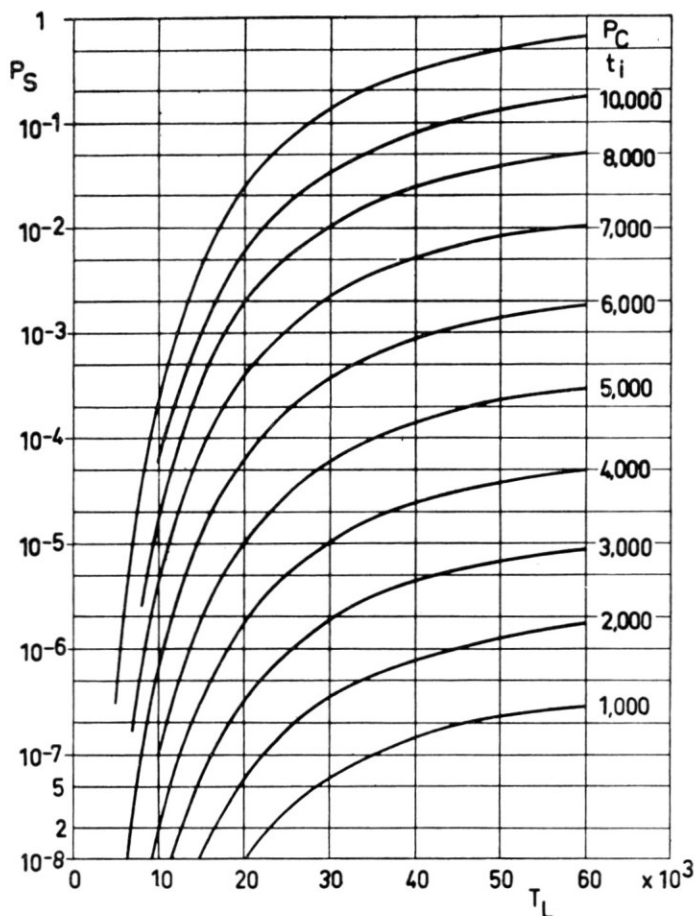


FIG. 4. Influence of length of inspection interval  $t_i$  on probability of collapse with increasing service life. The same parameters assumed as in Fig. 3.

corresponding safe-life structure ( $n = 1$ ). With increasing number of inspections the slope of the probability curves decreases but is still so appreciable that a limitation of the life of fail-safe structures seems, in principle, necessary.

When the service life and the number of inspection intervals are fixed the length of each inspection interval will also be known. By using the values of Fig. 3, a diagram has thus been drawn in Fig. 4 which gives



$P_S$  versus the service life  $T_L$  for various lengths of the inspection interval  $t_i$  from 1000 to 10,000 hours. This diagram gives a clear indication of the great importance of the inspection intervals. A shortening of the length, for instance, from 4000 to 1000 hours, will cause a reduction of  $P_L$  by a factor of over 100.

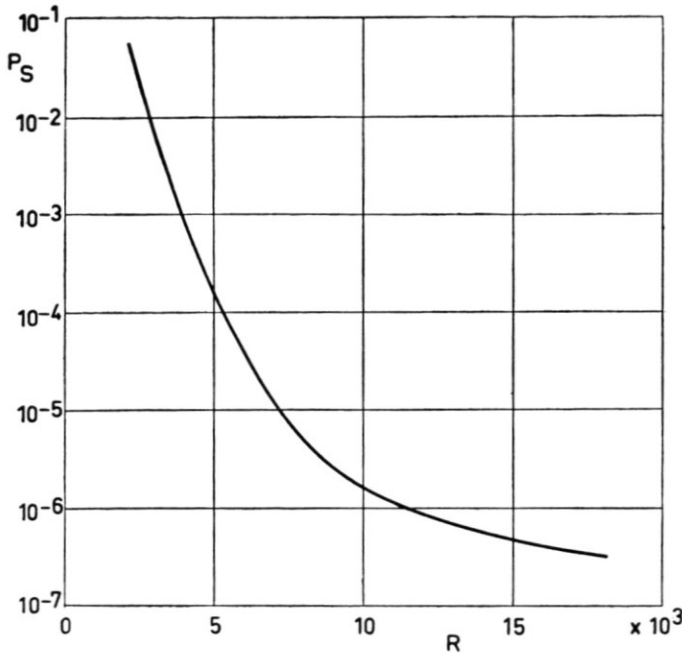


FIG. 5. Variation of probability of collapse with crack propagation parameter  $R$ .  
Other parameters assumed:  $T_{50}=50,000$ ;  $\sigma=0.20$ ;  $H_0=0.20$ ;  $h=20$ ;  
 $T_L=30,000$ ;  $t_i=3,000$ .

As is evident from section 3 of this chapter, the value of the crack propagation parameter  $R$  might vary considerably. In order to study the effect of an error in an assumed value of  $R$ ,  $P_S$  has been calculated for values from 2000 to 18,000 hours assuming the same parameters as for Figs. 3 and 4 and further  $T_L=30,000$  and  $t_i=3,000$  hours. The calculated points have been plotted in Fig. 5, which shows that the influence of a variation is quite considerable at low values of  $R$ , while the curve flattens out when  $R$  exceeds about 10,000 hours.

When designing an aircraft structure with respect to fatigue, the designer as said has mainly to resort to a variation of the stress level in order to attain the low probability of collapse required, if the service life is fixed and a lower limit has been put to the length of the inspection

intervals. A practical design example should consequently include the effect of the stress level. Figures 6 and 7 are intended to illustrate a phase of a realistic design procedure. As a first attempt no increase in the structural area is assumed. For a stress level thus corresponding to  $n_D = 3.75$  full-scale, and other tests might have indicated the following values for crack initiation in the wing section:

$$T_{50} = 25,000 \quad \sigma = 0.30$$

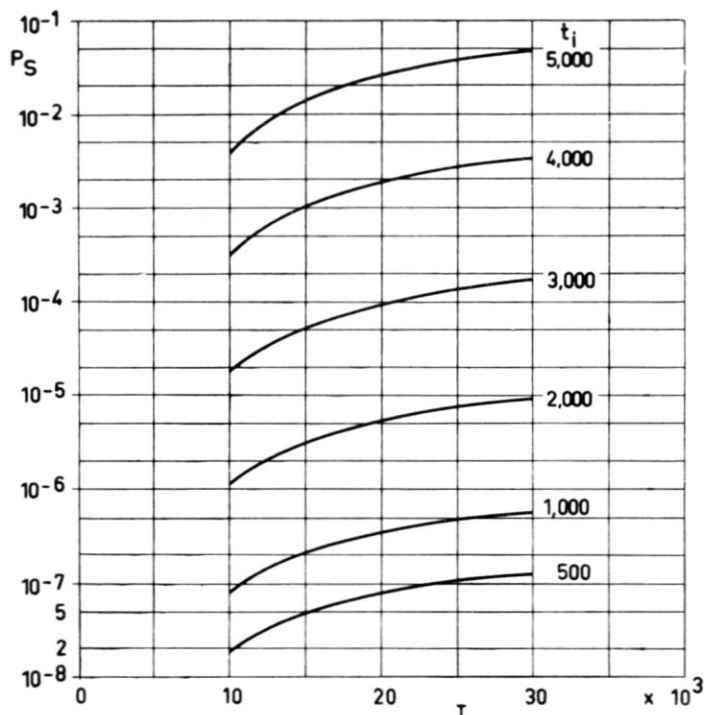


FIG. 6. Design example, first attempt with  $n_D = 3.75$  implying no margin in structural area for fatigue ( $k_A = 1.0$ ). Assumed parameters connected with this stress level:  $T_{50} = 25,000$ ;  $\sigma = 0.30$ ;  $R = 6000$ ;  $H_0 = 0.20$ ;  $h = 20$ .

A value of  $R = 6000$  hours is also supposed to have been determined experimentally at the same stress level. For the spectrum of heavy gust loads is assumed:

$$H_0 = 0.20 \quad h = 20$$

Figure 6 presents the probability of collapse  $P_S$ . Suppose now that the limit probability for the section has to be  $P_{LS} \leq 10^{-7}$  and that a limit life of at least 30,000 hours should be guaranteed. The figure reveals

that it would be necessary with this design to make inspections at intervals of less than 500 hours. Such a short inspection period would probably not be accepted by the operator. It is therefore necessary to decrease the stress level.

As a second attempt, the ultimate design load factor is augmented to 4.5 ( $k_A = 1.2$ ). Such a stress reduction can be estimated to yield an increase of fatigue life by a factor of about 3<sup>(18)</sup> giving  $T_{50} = 75,000$  hours, while the standard deviation might be assumed unaltered. The

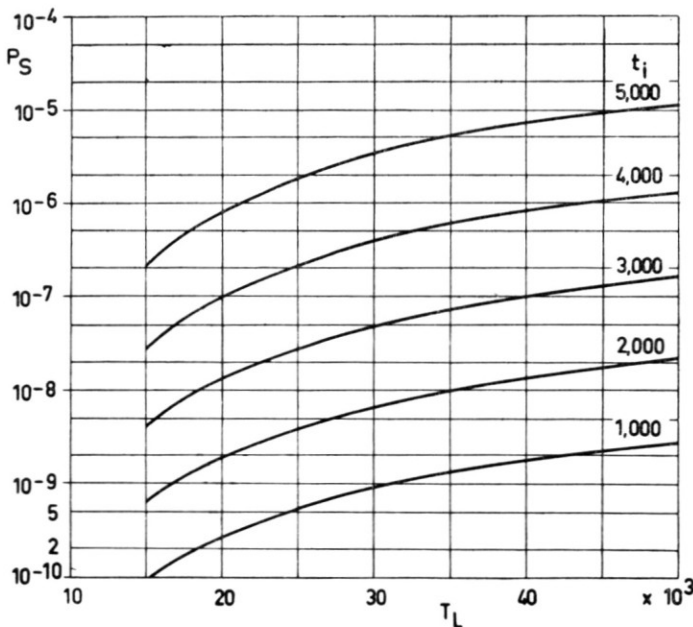


FIG. 7. Design example, second attempt with  $n_D = 4.5$ , ( $k_A = 1.2$ ). Compared with Fig. 6 this decrease in stress level is assumed to imply increases of  $T_{50}$ ,  $R$  and  $h$  to 75,000, 10,000 and 24 respectively, whereas  $\sigma$  and  $H_0$  are assumed to remain unaltered.

crack propagation parameter,  $R$ , will be increased to somewhere around 10,000 hours, i.e. by a factor of 1.7<sup>(14,15)</sup>. Finally, the spectrum parameter  $H_0$  is not affected, while  $h$  is increased by 20 per cent to a value of 24.

The resulting probability  $P_S$ , corresponding to  $n_D = 4.5$ , has been plotted in Fig. 7. A service life  $T_L = 30,000$  hours, with  $P_L \leq 10^{-7}$ , now corresponds to  $t_i = 3300$  hours, which might be an acceptable interval length. If a shorter or longer inspection interval is desired, a new diagram for a somewhat higher or lower stress level has to be prepared.

## REFERENCES

1. LUNDBERG, B., Fatigue life of airplane structures. The 18th Wright Brothers Lecture, *J. Aero. Sci.*, Vol. 22, No. 6 (June, 1955), p. 349-413. Published also by The Aeronautical Research Institute of Sweden as FFA Report 60 (1955).
2. LUNDBERG, B., Some proposals for evaluating fatigue properties of airplane structures. Proceedings of the Second European Aeronautical Congress, Scheveningen, The Hague, September, 1956. Published also by The Aeronautical Research Institute of Sweden as FFA Report 76 (1958).
3. LUNDBERG, B., Notes on the level of safety and the repair rate with regard to fatigue in civil aircraft structures. FFA Technical Note No. HE-794. Lecture to Eleventh Technical Conference of International Air Transport Association, Monte Carlo, September, 1958.
4. LUNDBERG, B., A statistical method for fail-safe fatigue design. FFA Technical Note No. HE-850, June 1959.
5. LUNDBERG, B., The quantitative statistical approach to the aircraft fatigue problem. FFA Technical Note No. HE-853. Lecture to ICAF-AGARD Fatigue Symposium, Amsterdam, 1959.
6. LUNDBERG, B., Draft PAMC-Fatigue Strength (presented by E. Ljungh). AIR C-WP/81, ICAO, Airworthiness Committee, Third Meeting, Stockholm, 14 July 1959.
7. KENNEDY, A.P., A method for determining the "safe" life of an aircraft wing from fatigue test results. *Journal of the Royal Aeronautical Society*, Vol. 58, No. 521, May 1954, p. 361-366.
8. FERRARI, R., Residual strength level and associated aspects of "fail safe" structures. Appendix I: Some considerations relating to the safety of "fail safe" wing structures, by R.M. Ferrari, I.S. Milligan, M.R. Rice, and M.R. Weston, Department of Civil Aviation, Australia. AIR C-WP/71, ICAO, Airworthiness Committee, Third Meeting, Stockholm, 14 July, 1959.
9. WHALEY, R.E., Fatigue investigation of full-scale transport-airplane wings. Variable-amplitude tests with a gust-load spectrum. NACA TN 4132, Nov. 1957.
10. FREUDENTHAL, A. M. and HELLER, R. A., On stress interaction in fatigue and a cumulative damage rule. *JA/SS*, July 1959, p. 431-422.
11. HARDRATH, H.F., LEYBOLD, H. A., LANDERS, CH. B. and HAUSCHILD, L. W., Fatigue-crack propagation in aluminium-alloy box beams. NACA TN 3856, August 1956.
12. PAYNE, A. O., FORD, D. G., JOHNSTONE, W. W., KEPERT, J. L., PATCHING, C. A. and RICE, M. R., Fatigue characteristics of a riveted 24S-T aluminium alloy wing. Part V. Discussion of results and conclusions. Report ARL/SM. 268, June 1959.
13. GROVER, H. J. BISHOP, S. M. and JACKSON, L. R., Fatigue strengths of aircraft materials axial-load fatigue tests on notched sheet specimens of 24S-T3 and 75S-T6 aluminium alloys and of SAE 4130 steel with stress-concentration factors of 2.0 and 4.0 NACA Technical Note 2389, June 1951.
14. MCEVILY, JR., A. J. and ILLG, W., The rate of fatigue-crack propagation in two aluminum alloys. NACA TN 4394, September 1958.
15. ILLG, W. and MCEVILY, JR., A. J., The rate of fatigue-crack propagation for two aluminium alloys under completely reversed loading. NASA TN-52, October 1959.
16. CRICHLow, W. J., The ultimate strength damaged structure. ICAF-AGARD Fatigue Symposium, Amsterdam, 1959.

17. COPP, M.R. and COLEMAN, T.L., Report of the Air Navigation Conference Doc. 7730, AN-CONF/3 Addendum Topic No. 2, ICAO, Montreal 1956.
18. LUNDBERG, B. and EGGWERTZ, S., The relationship between load spectra and fatigue life. The International Conference on Fatigue in Aircraft Structures, held at Columbia University, New York, January 30, 31 and February 1, 1956. Proceedings. Edited by A.M. Freudenthal. New York (1956), p. 255-277. Published also by The Aeronautical Research Institute of Sweden as FFA Report 67 (1956).

## APPENDIX A

### ASPECTS OF THE PROBABILITY OF COLLAPSE OF A CRACKED STRUCTURE

By LENNART VON SYDOW

In the treatment of the complex problem of fatal incident of an aircraft due to fatigue, several simplifying assumptions have to be introduced. The object of this paper is not to discuss whether certain assumptions are realistic but to deduce, in a purely mathematical way, a probability model based on the assumptions made.

The model is confined to one single section of a part of an aircraft, such as a wing. The combination of the models for the different sections is a different problem, which will not be considered here.

The varying probability of crack initiation as well as crack propagation and regular inspection are taken into account.

#### CRACK PROPAGATION

The first problem to be considered is to derive an expression for the probability of failure before time  $t$ , provided that there exists a crack which was initiated at time  $t = 0$ . This has been treated by D. G. Ford in (8, Appendix A) and on this account only a brief survey will be given here.

The probability of failure within the time interval  $(t, t+dt)$  is assumed to be proportional to the length  $dt$  of the interval with a factor  $\lambda(t)$  of proportionality which may vary with time. The probability  $P$  of failure before time  $t+dt$  may then be expressed by the following differential equation:

$$P(t+dt) = P(t) + (1 - P(t)) \lambda(t) dt \quad (1)$$

which means, in plain words, that the probability of failure before time  $t+dt$  is the sum of the probabilities of two mutually exclusive events:

1. failure before time  $t$
2. non-failure before time  $t$  and failure in  $(t, t+dt)$

The differential equation (1) may be written:

$$P'(t) + \lambda(t)P(t) = \lambda(t) \quad (2)$$

which may be solved by ordinary methods with the boundary condition

$$P(0) = 0 \quad (3)$$

By putting  $A(t) = \int \lambda(t)dt$  the solution may be written:

$$P(t) = 1 - e^{A(t) - \lambda(t)} \quad (4)$$

Under the same assumptions which were adopted in the paper mentioned above the function  $\lambda$  will be:

$$\lambda(t) = H_0 e^{-h\left(1 - \frac{t}{R}\right)} \quad (5)$$

The assumptions which lead to this are briefly that the number of gust loads that exceed the value  $s_t$  may be expressed by  $H_0 e^{-hs} t$  and that the loads  $s_t$ , necessary to fail the structure, decreases linearly with time so that  $s_t = 1 - \frac{t}{R}$ .

The solution is then easily found by insertion in (4):

$$P(t) = 1 - e^{\frac{H_0}{R} \frac{t}{h} e^{-h\left(1 - \frac{ht}{R}\right)}} \quad (6)$$

### *Crack Initiation*

The previous treatment assumes the existence of a crack. If no crack exists, the probability of failure is taken to be zero which implies that the case of failure of an intact section due to heavy gust is not treated.

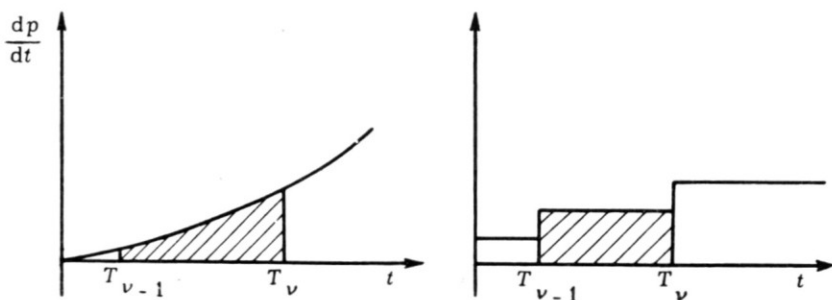
For the continuation it is now necessary to introduce a probability which bears upon the existence or the initiation of a crack. This may seem trivial but is most certainly not so. The initiation of a crack is a somewhat vague concept and cannot be directly measured. This implies that there are no ways of verifying a theoretical model by direct measurements because an infinitesimal crack is not measurable.

If the concept of initiation is replaced by the existence of the smallest crack detectable the conditions are vastly improved although a statistical observation material even on this event may be difficult to obtain. Moreover, one should bear in mind, that the assumptions for crack propagation and crack initiation have to be made with consistency, so that the parameters of the two models are chosen with respect to one another. In the extreme case of a crack being detectable only by total collapse of the whole section, it is obvious, that the crack propagation velocity must be taken to be infinite, i.e. instantaneous failure. On this account the origin of crack propagation has to be defined

with respect to some statistically measurable property of crack development, e.g. a given crack length attained.

Let now the probability of a crack occurring in the time interval  $(t, t+dt)$  be  $dp(t)$ . Then the probability of a crack occurring in  $(T_{v-1}, T_v)$  is:

$$\int_{T_{v-1}}^{T_v} dp(t) = \text{shaded area}$$



If no crack exists at time  $T_{v-1}$  the total probability of failure before time  $T_v$  may be calculated as follows:



Assume that a crack occurs at time  $t$ . The conditional probability of failure before time  $T_v$  is then evidently  $P(T_v-t)$  and the total probability of failure in  $(T_{v-1}, T_v)$  will be the integral from  $T_{v-1}$  to  $T_v$  of the joint probability:

$$\int_{T_{v-1}}^{T_v} P(T_v-t) dp(t) \quad (7)$$

### Inspection

The influence of inspection on the probability of failure is, of course, very much depending on what is carried out at an inspection. The following assumptions will be made:

1. An existing crack is always discovered and the part repaired.

This implies that crack propagation, if any, will be put to an end at the points of time of inspection.

2. The probability of a new crack occurring may or may not be influenced by the repairs.

By proper choice of the function  $dp$  the aging (increased probability of crack initiation with time) of the structure can thus be taken into account both without and with renewal of parts of the structure by repairs.

Let the inspections take place at

$$T_1, T_2, \dots, T_v, \dots, T_n$$

where  $T_n$  represents the limit service life. Using the expression (7), it is then obvious that the total probability of failure before time  $T_n$  is:

$$P_S = \sum_{v=1}^u \int_{T_{v-1}}^{T_v} P(T_v - t) dp(t) \quad (8)$$

It should be pointed out that the equation (8) is quite general and is only based on assumption 1. above. No specific forms of the functions  $P(t)$  and  $dp(t)$  have to be assumed to render it valid.

#### Numerical Evaluation

For the numerical evaluation of the function  $P_S$  it is suitable to make two additional assumptions.

1. The inspections are carried out at equidistant points of time; the inspection interval is denoted  $t_i$ .
2. The service life  $T_n$  is always a multiple of the inspection interval  $t_i$ .

Consider first for the function  $dp$  a large class of functions, the step-functions. The calculation of  $P_L$  will then be simple by putting

$$dp(T_v \leq t < T_{v-1}) = \frac{\pi_v}{t_i} dt$$

Hence

$$P_S = \sum_{v=1}^n \int_{T_{v-1}}^{T_v} P(T_v - t) \frac{\pi_v}{t_i} dt = \sum_{v=1}^n \pi_v \frac{1}{t_i} \int_{T_{v-1}}^{T_v} P(T_v - t) dt$$

Substituting

$$\tau = T_v - t \quad d\tau = -dt$$

will give

$$P_S = \sum_{v=1}^n \pi_v \frac{1}{t_i} \int_0^{t_i} P(t) dt$$

or since the integral is independent of  $v$

$$P_S = \frac{1}{t_i} \int_0^{t_i} P(t) dt \sum_{v=1}^n \pi_v = \overline{P(t_i)} P_C \quad (9)$$



Expression (9) may be interpreted as the product of the time average  $\overline{P(t_i)}$  of the function  $P(t)$  over the inspection interval  $t_i$  and the total probability  $P_C$  of a crack occurring during the whole limit service life.

It is interesting to notice that the step-function may have any form since only the total sum is involved in the result. It can thus easily be applied to actual measurements on certain types of aircraft.

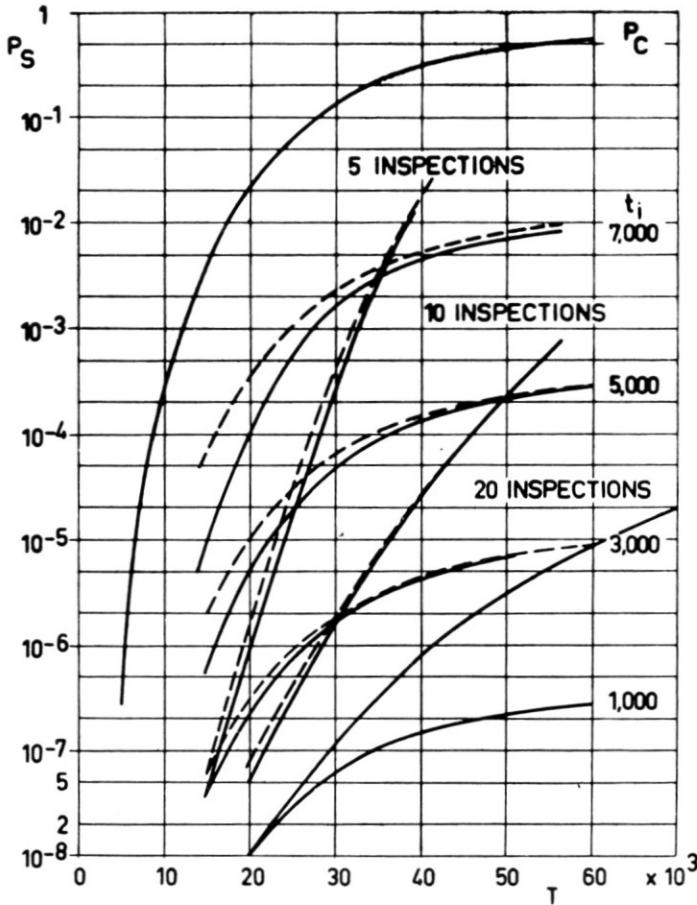


FIG. 1A.

If  $dp(t)$  is not a step-function, the form (9) may still be applicable as an approximation since a step-function is the most primitive form of approximation to an arbitrary function, the shorter the steps the better approximation. Then if the number of inspections is sufficiently large the equation (9) should tend to the same values as equation (8).

With the aid of an electronic digital computer the equation (8) has been evaluated with  $P(t)$  according to equation (6) and  $dp$  according to the logarithmico-normal frequency function

$$dp = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{(\log t - m)^2}{2\sigma^2}} dt$$

for different inspection intervals  $t_i$ . The result is plotted in Figure 1A of this Appendix. In addition the values of  $\overline{P(t_i)}$  have been computed and equation (9) has been evaluated with  $P_C$  according to the same logarithmico-normal distribution as above. The result is plotted in the same figure with dotted lines. Actually  $P_C$  should in this case be represented by a number of different polygons but this is of no practical importance.

#### DISCUSSION

J. F. CUSS: The author has quite rightly made a strong point of the need to match the residual airframe strength after cracking, with inspection periods which will ensure that risk of failure is kept within an acceptable limit. Is this however a theoretical approach in advance of the construction and fatigue test on the aircraft under consideration? When this test has been carried out under a proper programmed loading, the cracks will have been observed during the test and one has a very exact means for determining the inspection periods.

BO. LUNDBERG: In principle, the safest and most satisfactory way of showing compliance with a maximum probability of failure is by conducting a sufficient number of full-scale tests of the fatigue-prone components, or sections, for instance of a wing. It is my belief, however, that when an appreciable amount of information from tests of various types of aircraft designs, in particular wing structures, has been compiled, the designer will be able to apply with due conservatism "reduced static strength functions" which are fairly representative for the intended type of design. Only then will it be possible to develop a design on the basis of theories of a type exemplified by the paper without going to the expense of a considerable number of full-scale tests prior to design finalization. Whether or not one or a few confirmative full-scale tests with the completed design will be needed for satisfying the airworthiness authorities, is a question that can hardly be answered at this stage in a general way.

I wish to point out, however, that it does not seem to be possible to determine appropriate inspection intervals merely on the basis of observed cracks in tests under a proper programmed loading. The determination of safe inspection intervals can only be achieved by combination of test results with an appropriate statistical theory.

E. D. KENN: It is my opinion that this is a most outstanding and important paper in that it forms what is probably the first basis for a real understanding of fail-safe structures. It will probably be a long time before airworthiness authorities accept the authors' proposals but they form a very desirable goal. There are, however, two important reservations with respect to the premises laid down. The first one is mentioned by the author and that is the fact that the effect of crack propagation on static residual strength can be non-linear. This may not affect the method but it could affect the result very significantly. Secondly, the author does not consider the possibility of more than

one crack occurring at the same cross-section. In a well designed structure it is considered that such a possibility cannot be ignored, and the authors' remarks would be appreciated.

BO. LUNDBERG: I wish to thank Mr. Keen for his encouraging appreciation of the paper. Mr. Keen is quite right in his two observations that the effect of a non-linear static residual function might be quite significant and that the possibility of more than one crack occurring at the same cross-section within one inspection interval has to be considered. As a matter of fact I can state that these very two questions are the subject of extended research presently under way at the FFA.

B. J. LAZAN: Mr. Lundberg has very effectively demonstrated the significant increase in structural fatigue life realizable through frequent inspection. Implicit in his analysis is, of course, the assumption that the inspection methods used will detect the fatigue damage and cracking caused by prior service. In the case of structures with significant service records prior experience helps to localize fatigue-prone regions so that access openings and special inspection methods may be selected to be sensitive to anticipated cracking patterns. In the case of new structures, however, the probability of detecting localized fatigue damage by general inspection methods is likely to be rather low. Thus, intuitive feelings regarding where to expect trouble must often be relied upon. Would Mr. Lundberg comment on: (1) The probability of predicting the locations of likely fatigue damage in new structures, and (2) If the probability is sufficiently low in new structures so as to prevent the fatigue problem from being serious until some service experience can be assembled.

BO. LUNDBERG: The observations made by Professor Lazan are indeed important. As a matter of fact, they boil down to the question of what can in practice be defined as "detectable length" of a fatigue crack. Obviously, the "detectable length" depends on quite a few conditions, such as accessibility of the structure to inspection, for instance by means of access openings, to which extent more sophisticated inspection methods are employed, such as X-ray, etc. In principle, service experience is also of importance because if the inspectors know where cracks are likely to appear, they should normally be able to detect shorter cracks than if no previous experience has been accumulated. It is, therefore, quite logical to modify the definition of "detectable length" of fatigue cracks as experience has been gained in such a way that the "detectable length" is made successively shorter than would be safe to assume for a new structure.

In reality, however, some caution should be exercised in this respect for a number of reasons, the main one being that it might direct too much attention to the structural regions which are believed to be fatigue-sensitive at the expense of other regions which, although less fatigue-sensitive, might also be less accessible. Another reason is that a modification, in the course of service life, of the "detectable length" would, of course, complicate the statistical theory.

The two specific questions that Professor Lazan has raised, might be commented on as follows:

(1) The possibility of predicting the locations of fatigue damage in new structures is mainly dependent on the completeness of the laboratory tests which have been conducted before deliveries of the aircraft. In particular it might be pointed out that if the investigations are conducted using full-scale tests, with the various fatigue-sensitive components or sections, using quite realistic load spectra, and if, furthermore, such tests are carried through well beyond the anticipated "reference" limit life of the aircraft, then there should be very good chance of predicting the location of fatigue cracks. Perhaps I might add that the possibility of predicting fatigue damage does not by itself prevent the possibility of structural collapse, because a crack might reach such a length before the next inspection that the structure can be fractured due to a heavy gust, as shown by the paper.

(2) To the question whether the probability might be sufficiently low in new structures so as to prevent the fatigue problem from being serious until some service experience has been assembled, I might refer to my previous general comments. I should add that as the probability of crack initiation is, normally, rather low in the beginning of the service life, the most efficient way of distributing a certain number of inspections during the service life should be by making the inspections more and more frequent towards the end of the service life. As the manufacturers seem to prefer constant inspection intervals, it is obvious that the probability of collapse in the "reduced static strength case" is rather low in, say, the first half of the service life. Under this condition the fatigue problem would, in principle, not be very serious in the beginning of the service life. On the other hand I ought to warn against complacency with regard to possible fatigue damage in new structures, as unforeseen damage can well appear at an early stage due to the possibility that the laboratory tests, in spite of all efforts, have not been quite realistic with regard to simulating the actual loading conditions.